

Nonlinear Shock Wave Propagation in Soft Tissues as a Mechanism of Injury

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Abstract

The aim of this study is to investigate the propagation of shock waves and self-preserving waves in soft tissues such as aorta and brain as a mechanism of injury in high rate loading conditions as seen in blunt trauma and blast-induced trauma (BIT). It is shown that such phenomena can only be seen in nonlinear viscoelastic materials and the existing linear and quasi-linear models predict only decaying waves. Various attempts to explain the mechanisms of soft tissue injuries e.g., traumatic aortic rupture (TAR) and traumatic brain injury (TBI) as a result of car accidents and sports injuries have been reported over the past 3 decades. In recent years, with advances in protective gears, blast induced trauma (BIT) has also become a major concern. To date, the mechanisms of soft tissue injuries, especially at high-rate loadings, are still not clearly understood.

As a blast wave enters a biological tissue, high-rate stress waves, which have longitudinal and shear components, develop in the tissue and such components can have devastating effects on the tissue based on the amplitudes of the waves and the orientation of tissue fibers. In this study, nonlinear viscoelastic wave propagation in soft tissues is studied and a criterion for the development of one-dimensional shock waves has been proposed. It is shown that realistic jumps in the acceleration that may happen in blast or blunt trauma evolve to shock waves that result in large discontinuities in strain and stress that may lead to tissue failure.

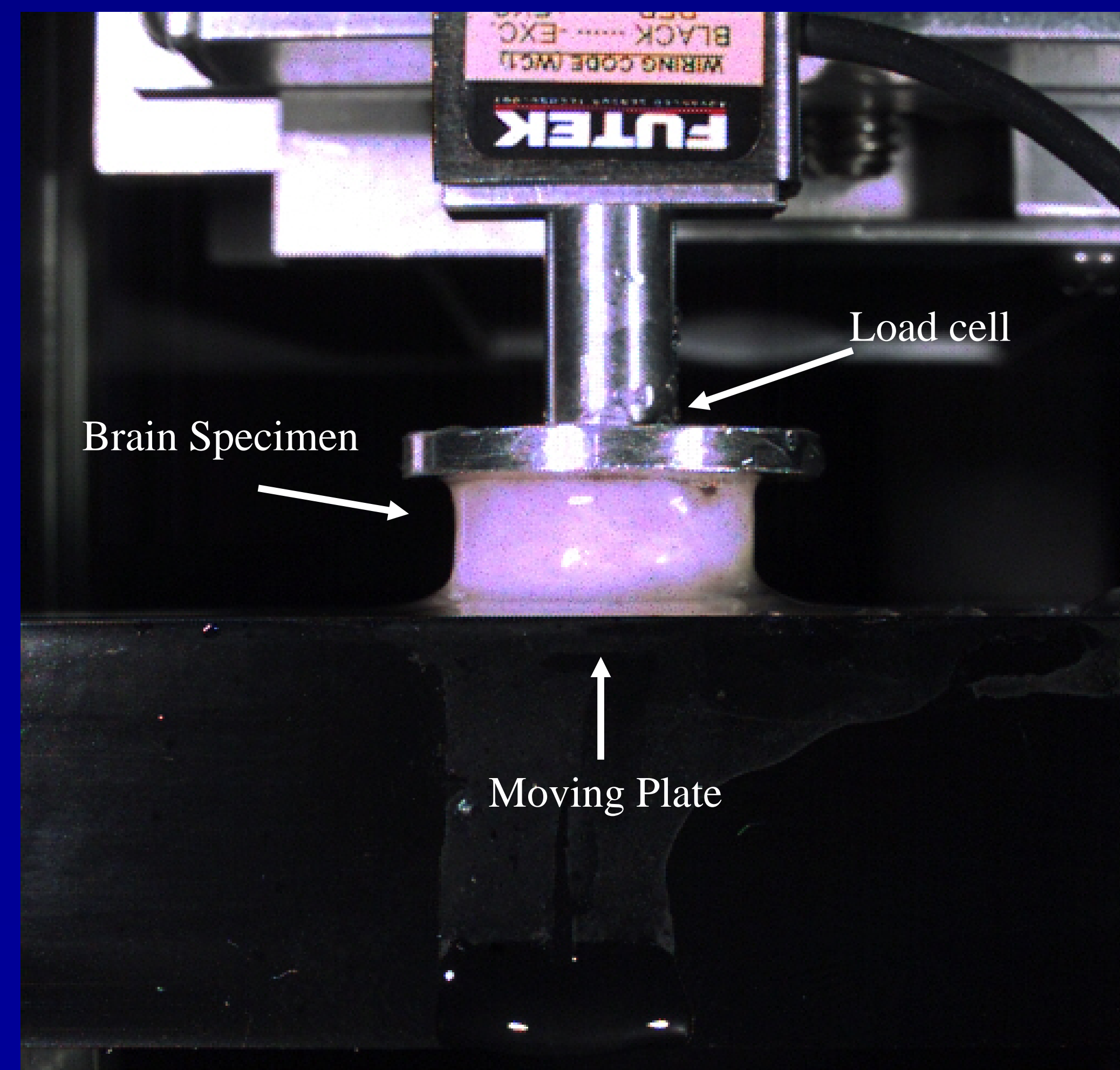


Figure 1 - The experimental setup. Brain specimen is placed between two plates undergoing compression. Sample can expand in the direction perpendicular to the motion

Objectives

The objective of this study is to first determine a physically viable material model for brain tissue, and to show the possibility of formation of shock waves in such material. Secondly, the propagation of shock waves and self-preserving waves in soft tissues has been studied and such waves have been proposed as a mechanism of injury in soft tissues.

Uniaxial Experiments

- Cylindrical samples of brain tissue with diameter~13mm and height~9mm cut
- Samples wetted on the top and bottom surface to allow free boundary condition (Fig. 1)

Theoretical Background

Hyperelastic Model

- Generalized Mooney-Rivlin model was used (6 terms)

$$W = \sum_{p,q=0}^N C_{pq} (I_1 - 3)^p (I_2 - 3)^q$$

- Coefficients derived from two isochronous set of data, one close to instantaneous response and one close to steady-state response
- Coefficients optimized using a constant k_0 for true instantaneous and steady-state response

Viscoelastic Model

- Quasi-Linear Viscoelastic (QLV) model used
- G_i and β_i are reduced relaxation amplitudes and decay rates respectively.
- Four decay rates chosen [100, 10, 1, 0.1]

$$S(t) = \int_{-\infty}^t G(t-\tau) \frac{\partial S^e}{\partial E} \frac{\partial E}{\partial \tau} d\tau$$

$$G(t) = G_{\infty} + \sum_{i=1}^4 G_i e^{-\beta_i(t-\tau)}$$

$$S^e(E) = k_0 f(E)$$

$$f(E) = E + f_2 E^2 + f_3 E^3 + \dots$$

Shock Wave Propagation

- Plain longitudinal motion in homogeneous material
- Intrinsic velocity ($U(t)$) defined as the time-derivative of the trajectory of the material point where the wave front is located at.
- Jump in parameter f is the change in its value immediately behind and after the wave front

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$$[f] = f^-(t) - f^+(t)$$

- Then we have

$$\frac{d[f]}{dt} = \left[\dot{f} \right] + U \left[\frac{\partial f}{\partial X} \right]$$

- For a wave entering a homogeneous region initially at rest, the jump in the particle's acceleration is obtained from the ODE below

$$a(t) = \frac{a_c}{\left(\frac{\lambda}{a(0)} - 1 \right) e^{\eta t} + 1}$$

where

$$\eta = \frac{-G'(0)}{2} = cte, \quad a_c = \frac{G'(0)}{2} \sqrt{\frac{k_0}{\rho_0}} = cte$$

In the above equations, a_c acts as the critical amplitude and is a material constant. This means that for a given material, there exists a critical amplitude, which determines that when a discontinuous wave enters the material, which is a characteristic of blast waves, whether the wave will be damped and dissipated or continue as an acceleration wave with constant amplitude or even become a shock wave, in which case it can have devastating effects on the tissue in terms of injury.

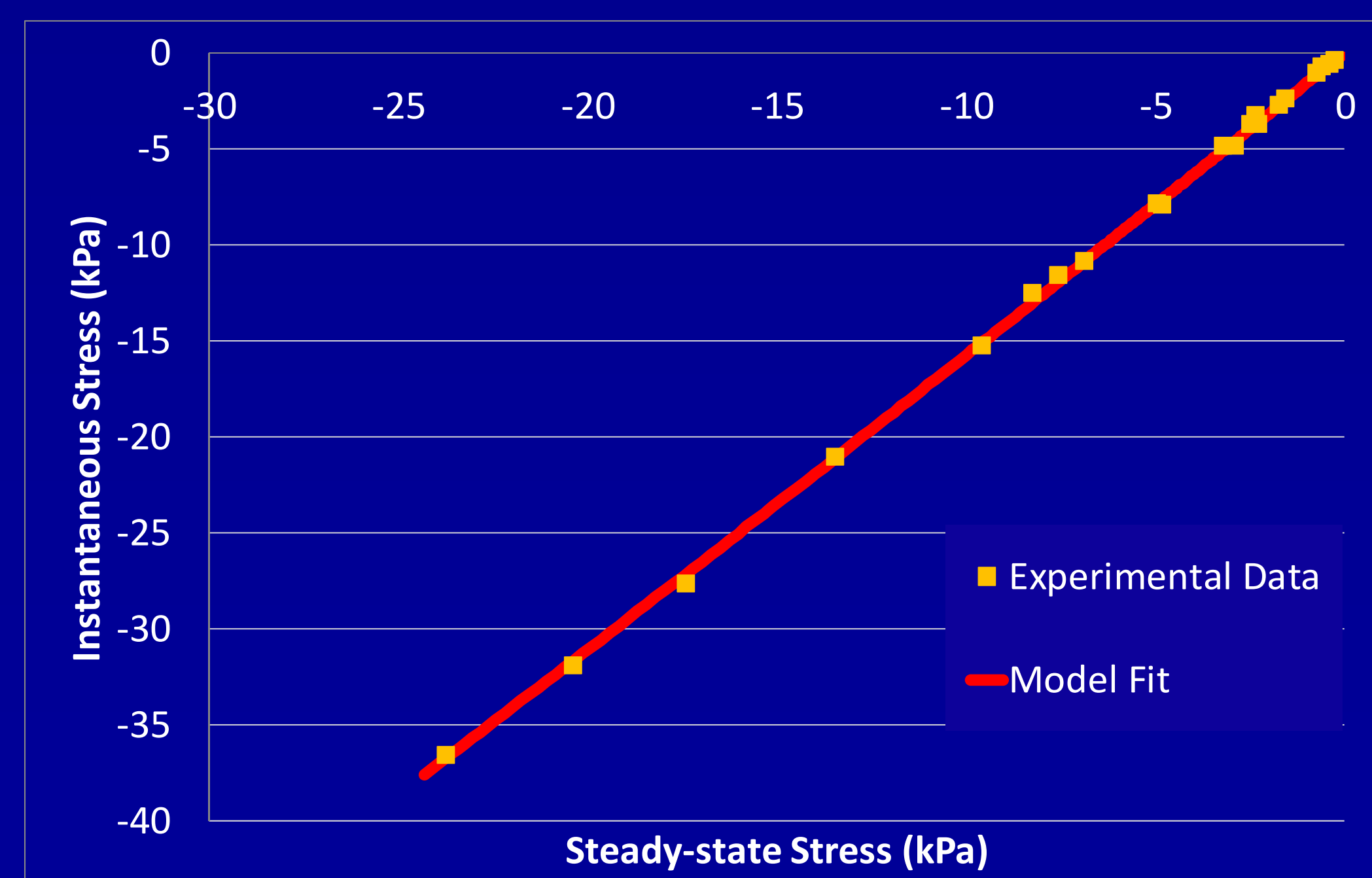


Figure 2 - The ratio between the instantaneous elastic response and the steady-state response

Results

As shown in Fig. 2, the ratio between the instantaneous elastic response and the steady-state response is very close to constant and therefore the QLV is an acceptable material assumption. Otherwise, two separate functions were required for instantaneous and steady-state responses to model the behavior of the material.

Mooney-Rivlin's elastic coefficients derived from instantaneous and steady-state responses (Fig. 3). The coefficients are determined through a least-squares optimization Viscoelastic parameters determined from stress-relaxation curves (Fig. 4). The relaxation function used as the model is fitted to the stress-time data from the experiments while decay rates have been chosen initially based on the ramp time and duration of the experiments

A significant aspect of the current study is the possibility of formation of shock waves inside the material as shown in Fig. 5.

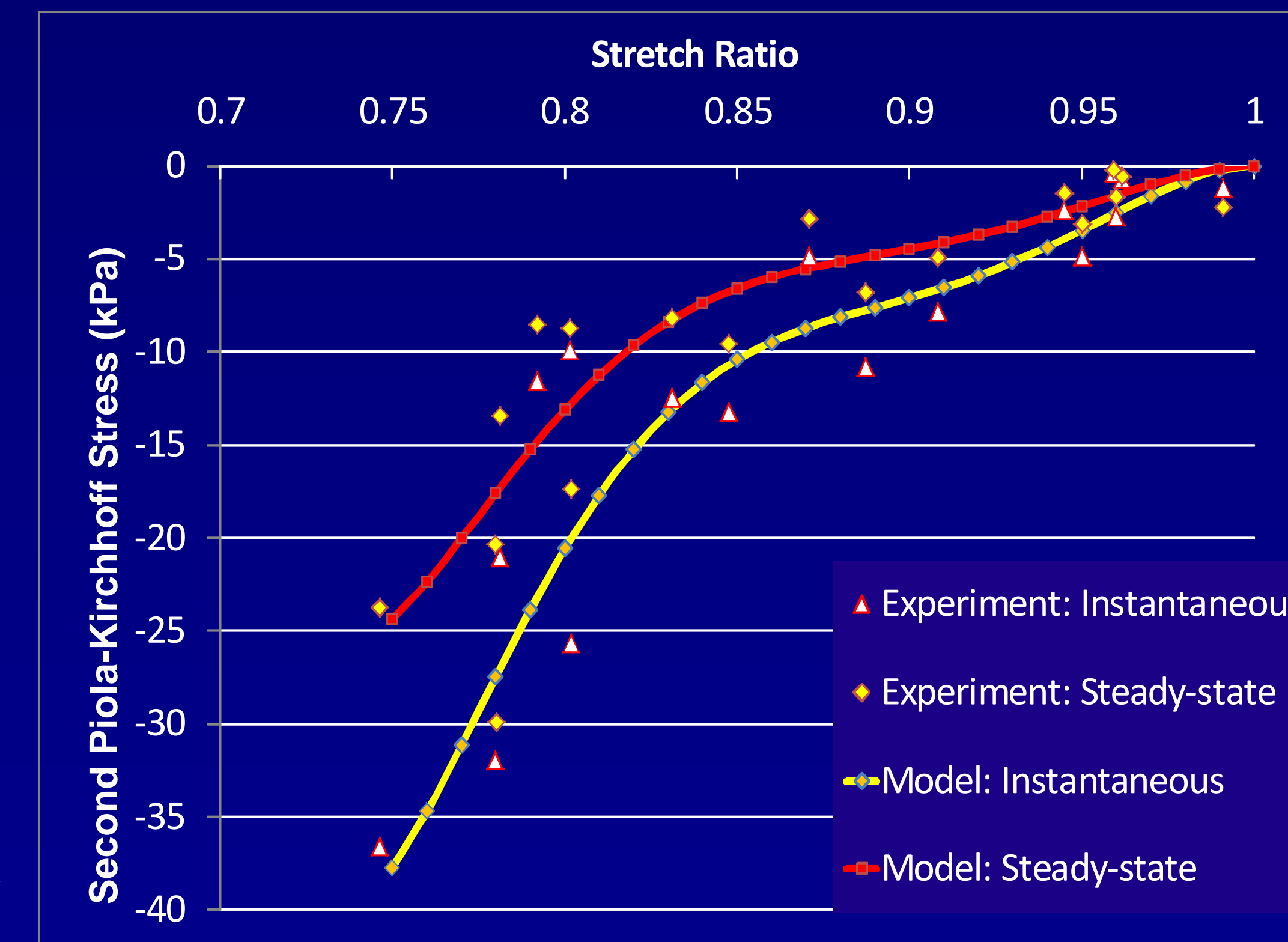


Figure 3 - Instantaneous and steady-state elastic functions fitted to the experimental data in the form of generalized Mooney-Rivlin hyperelastic solid

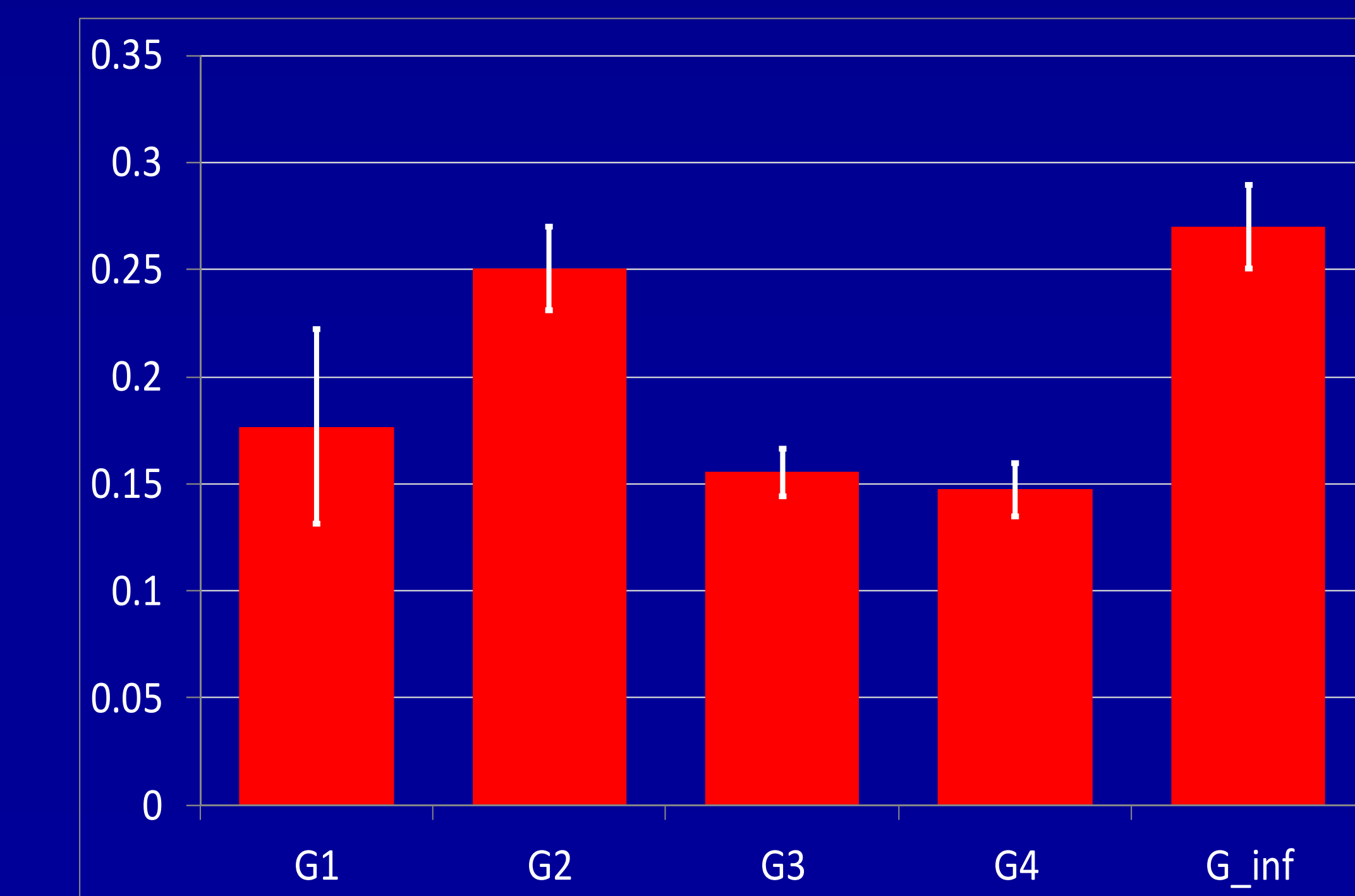


Figure 4 - The reduced relaxation coefficients (G_i)

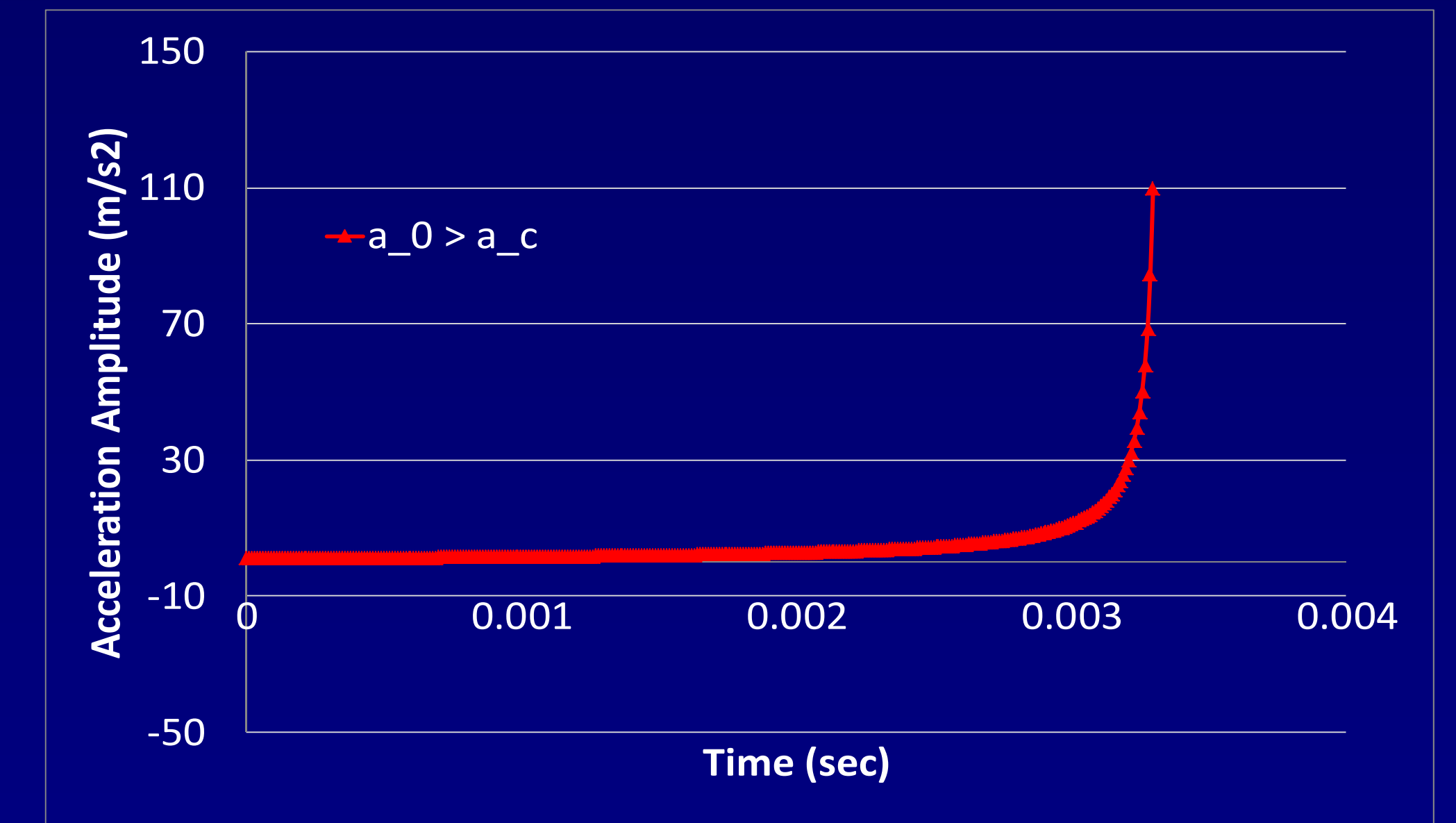
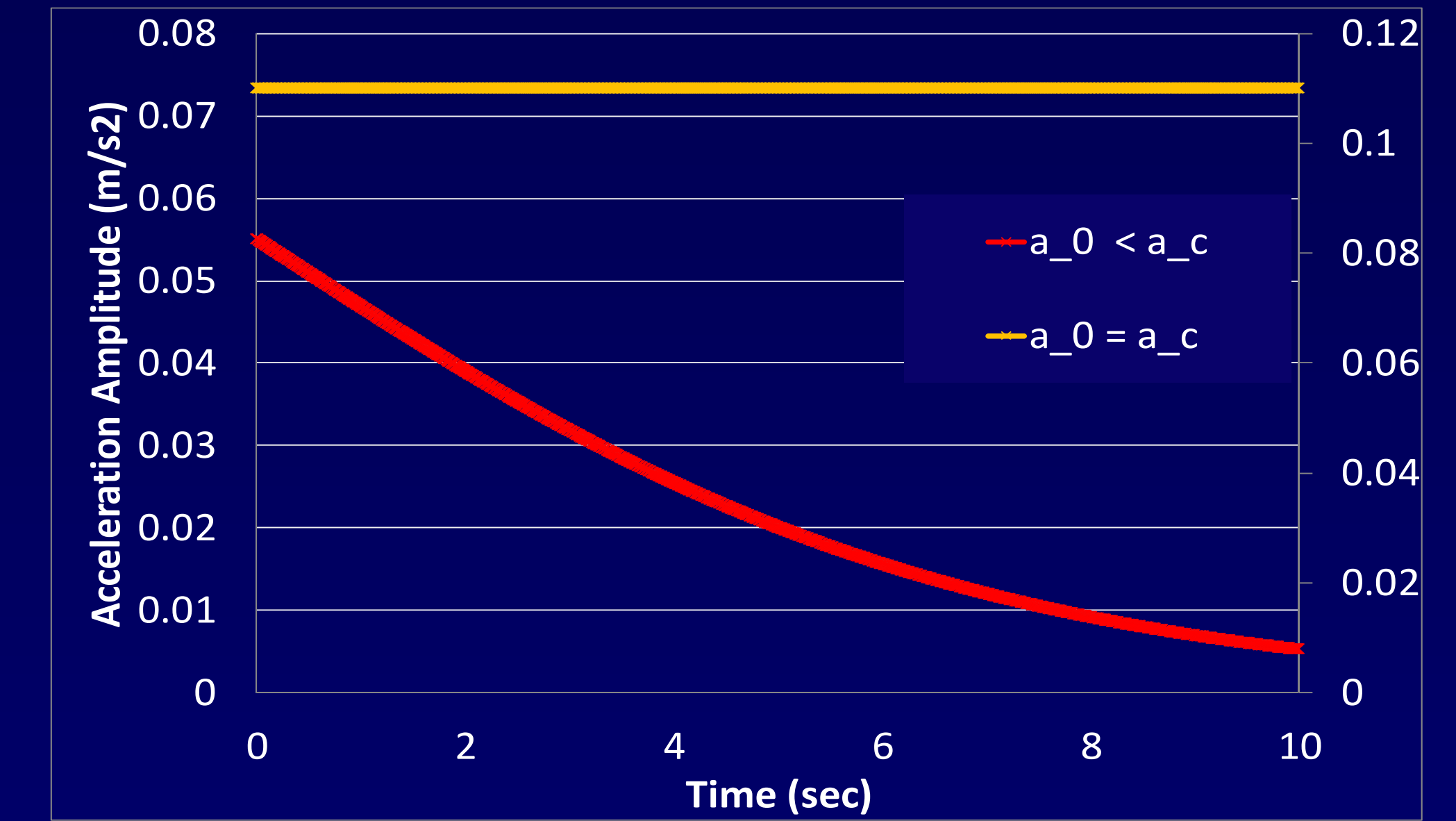


Figure 5 - Formation of shock waves. Based on the initial magnitude of the amplitude of the wave, it can be damped or form into a shock wave.

Conclusions

A novel method has been used in order to determine the material parameters. In this model, the elastic coefficients were derived from isochronous curves. This eliminates the need for non-physical data fitting.

Based on the equations above, we can see that the initial value of the discontinuity in acceleration plays a significant role in determining whether a shock wave will form or be damped (Fig. 5).

Acknowledgement

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References

- [1] Nyein, N., Jerusalem, A., Radovitzky, R., "Modeling Blast Related Brain Injury", 26th Army Science Conference (2008).
- [2] Spence, D., "Nonlinear Wave Propagation in Viscoelastic Materials", *Proceedings of a symposium conducted by the Mathematics Research Center* (1973), pp 365-397.
- [3] Coleman, B., Gurtin, M., "Waves in Materials with Memory", *Archives of Rational Mechanics* (1965), Vol. 19, 239-265.
- [4] Miller, K., Chinzei, K., "Constitutive Modeling of Brain Tissue: Experiment and Theory", (1997), *J. Biomechanics*, Vol. 30, 1115-1121.