LUBRICATION THEORY BASED MODELING OF CEREBROSPINAL FLUID DURING HEAD IMPACT

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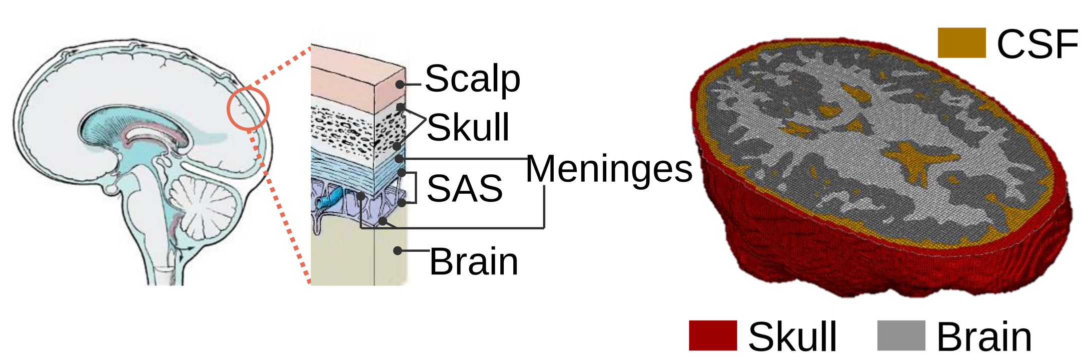
Introduction

(A) Anatomy of the brain skull interface [1] (left) and axial view of FE mesh of the head (right) [2].



BRAIN-SKULL INTERFACE: MENINGES + SAS

- Subarachnoid space (SAS): A chamber containing cerebrospinal fluid (CSF) & trabeculae
- Meninges: 3 membranes between brain & skull.

The interface cushions the brain during an impact by :

- Allowing for relative motion between brain & skull
- Reducing mechanical stresses & strains on the brain

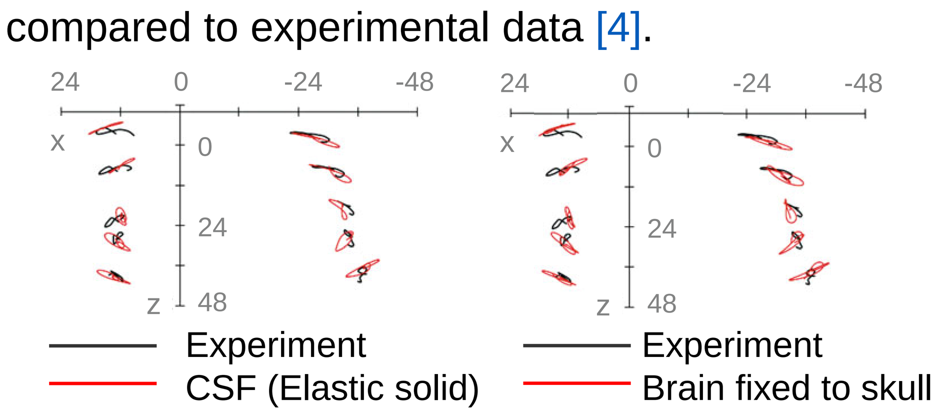
Several finite element head models (FEHM) have been developed in literature to study brain injury mechanisms due to impact. A major source of discrepancy among the FEHMs is the choice of constitutive model for CSF[3-5]:

- Solid: Elastic, Viscoelastic, Hyperelastic
- Simplified fluid: Hydrostatic fluid, Inviscid fluid, Mie Grunsein equation of state (EOS)

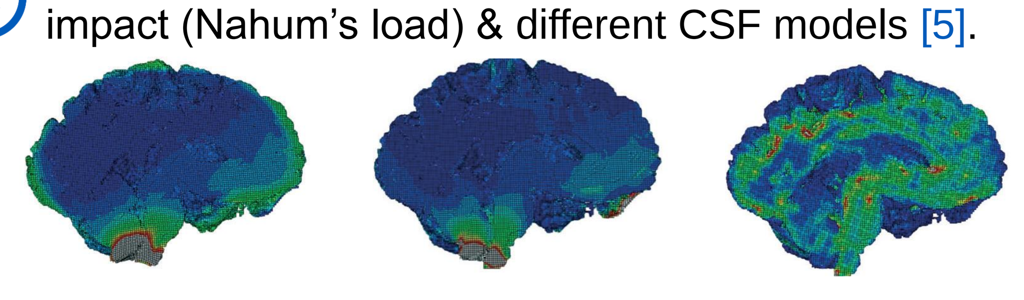
The drawbacks of the aforementioned models are:

- Excessive prediction of the brain displacement [Fig. B] and variability in the stress wave propagation in the brain [Fig. C]. Wide ranges of constitutive parameters reported in literature [3-5].
- SAS requires a fine mesh across its thickness [Fig. A]

(B) Relative motion of brain w.r.t skull predicted by THUMS FEHM for different CSF interface models compared to experimental data [4].



(C) Von-Mises stresses in the brain at t=5.5 ms for frontal impact (Nahum's load) & different CSF models [5].



Background

Several experimental works [6-7] have used surrogate head models to observe significant flow of CSF at the brain-skull interface during impact. Hence it is essential to study the role of CSF flow in injury prevention and mitigation.

Lubrication theory is a simplification of Navier-Stokes (N-S) equations which assumes that the liquid flow occurs in a narrow channel between two moving surfaces. Clearly, it can be employed for the CSF in SAS (thickness ~ 1mm) .

Objective: To develop a lubrication theory based model of the CSF flow in the SAS during an impact to the head.

Methods

1. Relevant parameters of the flow were used to determine the order of important non dimensional ratios [Table 1]. These ratios were used to scale N-S and continuity equations to obtain a simplified system of equations [Eqs. (1)-(4)] that govern the CSF flow in SAS.
2. A new FE CSF model was developed to compute the CSF velocity & pressure for a prescribed brain & skull motion [Eqs. (5)-(7)].

(D) Schematic of head geometry, coordinate system & an element of the novel FE model used in this work.

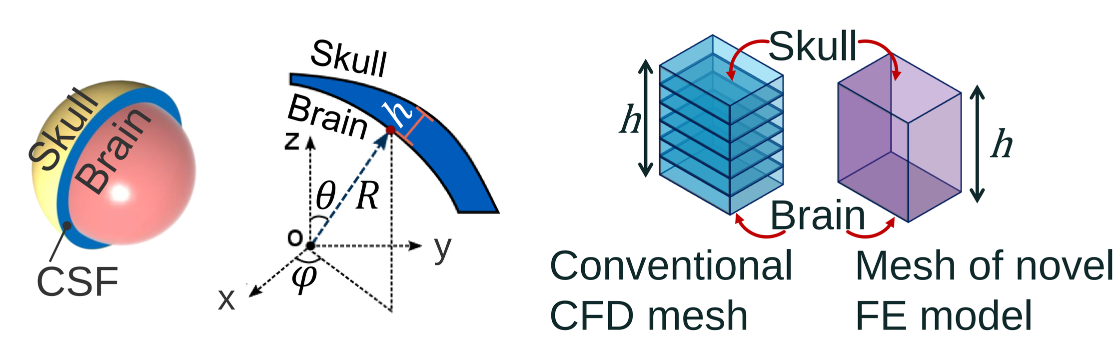


TABLE 1 : NOMINAL VALUES OF PARAMETERS

SAS thickness (h^*)	1 mm	Density (ρ)	Viscosity (μ)
Brain radius (R^*)	70 mm	1150 kg/m³	0.24 Pa.s
Radial velocity (v_0)	1 mm/s	CSF+ trabeculae [8]	
Diffusion time $\left(T_d = {}^{\rho h^{*2}}/_{\mu}\right)$: 10 ms		Impact time (T)	15 ms
SCALING PARAMETERS [9]			
Aspect ratio	Reynolds number	Temporal ratio	
$\epsilon = \frac{h^*}{R^*} \ll 1 \quad (a)$	$Re = \frac{\rho v_0 h^*}{\mu} \ll 1 \quad (b)$	$\frac{T_d}{T} \sim 1 \quad (c)$	

LUBRICATION THEORY WITH TEMPORAL INERTIA (NOVEL CONTRIBUTION)

$$(1)\quad \frac{\partial h}{\partial t} + \frac{1}{R\sin\theta}\left\{\frac{\partial\left(h\overline{v_\theta}\sin\theta\right)}{\partial\theta} + \frac{\partial\left(h\overline{v_\varphi}\right)}{\partial\varphi}\right\} = 0$$

Continuity Eq. averaged along r

$(\overline{v_\theta}, \overline{v_\varphi})$: Average velocities along r

$$(2)\quad \frac{\partial p}{\partial r} = 0$$

$$(3)\quad \rho\frac{\partial v_\theta}{\partial t} = -\frac{1}{R}\frac{\partial p}{\partial\theta} + \mu\frac{\partial^2 v_\theta}{\partial r^2}$$

$$(4)\quad \rho\frac{\partial v_\varphi}{\partial t} = -\frac{1}{R\sin\theta}\frac{\partial p}{\partial\varphi} + \mu\frac{\partial^2 v_\varphi}{\partial r^2}$$

Navier-Stokes Eqs. scaled using (a-c).

Eq.(1) is independent of r, Eq. (2) establishes that pressure is constant along r and Eqs.(3-4) are 1D parabolic PDEs in r for a given (θ,φ). Eqs.(3-4) are solved for a given pressure $p(\theta,\varphi,t)$ using a Fourier series approach. Hence, the average velocities $(\overline{v_\theta}, \overline{v_\varphi})$ are obtained as :

$$(5)\quad \overline{v_\theta}(t;\theta,\varphi) = \frac{v_{s\theta}+v_{b\theta}}{2} + \int_0^t \Phi(\tau,t)\left(-\frac{1}{\rho R}\frac{\partial p}{\partial\theta} - \frac{a_{s\theta}+a_{b\theta}}{2}\right)\Bigg|_{\theta,\varphi} \mathrm{d}\tau$$

Brain & Skull interface velocity & acceleration components

$$(6)\quad \overline{v_\varphi}(t;\theta,\varphi) = \frac{v_{s\varphi}+v_{b\varphi}}{2} + \int_0^t \Phi(\tau,t)\left(-\frac{1}{\rho R\sin\theta}\frac{\partial p}{\partial\varphi} - \frac{a_{s\varphi}+a_{b\varphi}}{2}\right)\mathrm{d}\tau$$

where $\Phi(\tau,t) = \sum_{n}^{odd} \frac{8}{n^2\pi^2}\exp\left(-\int_\tau^t \frac{\mu n^2\pi^2}{\rho h^2}\mathrm{d}\hat{t}\right)$ Convolution integral

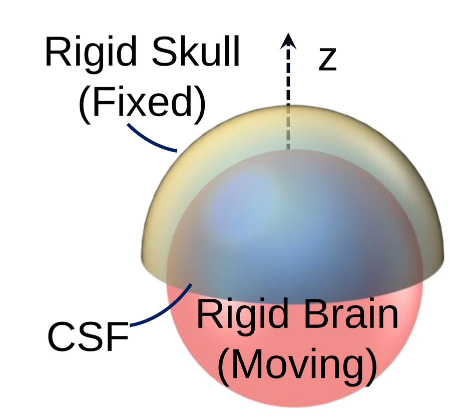
Next, Eqs.(5-6) are substituted in Eq. (1), resulting in an integro-differential equation for the pressure which is solved numerically using a novel FE model.

$$(7)\quad \underbrace{\mathbf{F}(t)}_{\text{Nodal load}} = \int_0^t \underbrace{\mathbf{K}(t,\tau)}_{\text{Stiffness matrix}}\underbrace{\mathbf{P}(\tau)}_{\text{Nodal pressure}}\,d\tau$$

System of FE equations solved incrementally using numerical integration

Results

(E) Brain translating w.r.t skull in z direction



Geometry & B.C.

R_b=70 mm

$h(t=0)$=3 mm

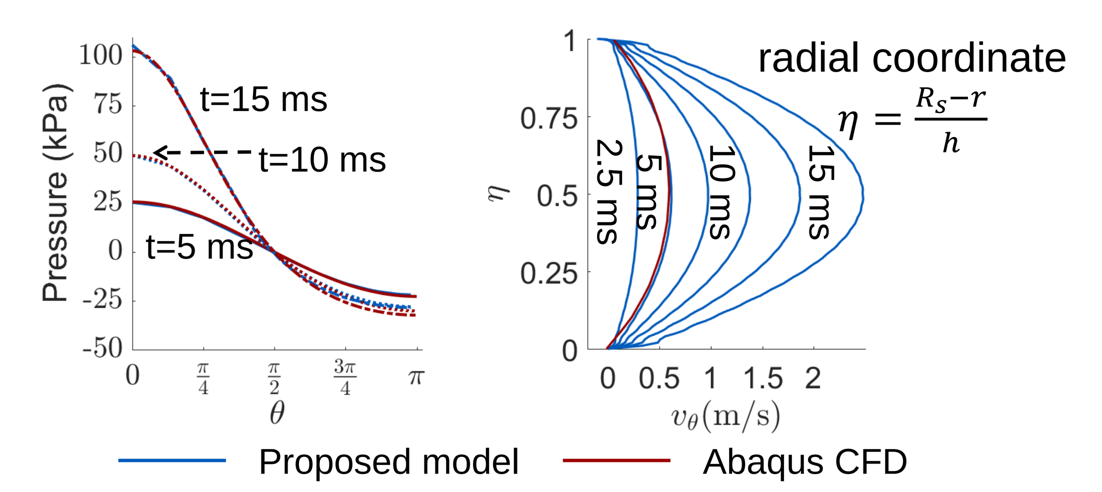
$\boldsymbol{v}_s(t) = \mathbf{0}$

$v_{bz}(t) = (10\,{}^{\mathrm{m}}/_{\mathrm{s}^2})t$

$p(\theta = {}^{\pi}/_{2}, t) = 0$

The proposed FE model is implemented in *Matlab* (*R 2017b*). The simplified geometry & B.C. data [Fig. E] are used to validate the novel FE model using *Abaqus CFD* (*6.14-5*).

(F) Comparison of pressure (left) and velocity profile (at $\theta = {}^{\pi}/_{4}$) obtained from both models at different time instants.



	Proposed model	Abaqus CFD
Domain	$\{\theta \in [0,\pi]\}$ (axisymm. flow)	3D Shell of thickness 3 mm
Mesh	Stationary 1D linear mesh ($N_e = 70$)	Moving 3D Linear brick mesh ($N_e = 51600$)
Cost	16 s	300 s

DISCUSSIONS

- Excellent overall agreement b/w proposed & CFD model for both velocity & pressure profiles. About 10% error between the pressure profiles observed at contrecoup location at larger times.
- Coup pressure increases at higher rate as compared to contrecoup [Fig. F, left] as seen in experimental tests of translational head impact. Transient squeezing effects also observed [Fig. F, right] [9].

Conclusion

A novel computationally efficient FE model has been developed to compute the CSF flow in the brain skull interface. A coupling algorithm is currently being devised to establish the interactions between CSF flow and brain deformations during an impact to the head

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