

A Multi-Stage Inverse Finite Element Method for Determining the Constitutive Model for Human Heel Pad Under High Rate Axial Loading

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Introduction

- ❖ Previous analysis using a Finite Element (FE) model of the leg has demonstrated that the nonlinear and viscoelastic properties of the human heel pad significantly influence the force transmitted to the tibia in high-rate axial compression [1].
- ❖ Suitable closed-form analytical solutions do not exist for determining the constitutive relationship of the materials tested, due to complex boundary conditions, sample dimensions, and inertial effects.
- ❖ Inverse Finite Element analysis (iFE) can be used to determine constitutive models from mechanical testing with no analytical solutions, but is limited by the large computational costs associated with optimization.

Objectives

1. Develop a multi-stage iFE method where nonlinear and viscoelastic material parameters are optimized for individual tests over their effective time domains
2. Apply this method to a FE representation of the heel pad to determine its Quasi-Linear Viscoelastic (QLV) constitutive model
3. Investigate differences in the constitutive model for iFE and previously determined analytical methods [1].

Constitutive Model

Quasilinear Viscoelasticity Framework [1]

The stress response $\sigma(\epsilon, t)$ to finite strain $\epsilon(t)$ was modeled using a QLV framework [1,3].

$$\sigma(\epsilon, t) = \int_0^t G(t-t') \frac{\partial \sigma^e(\epsilon)}{\partial \epsilon} \frac{\partial \epsilon}{\partial t'} dt'$$

Strain Energy Density (SED)

$$W = \frac{\mu}{2\beta} [e^{\beta(I_1-3)} - 1]$$

Instantaneous Elastic Response (IER)

$$\sigma^e(\lambda) = \frac{\mu_0 e^{[\gamma(\lambda^2 + \frac{2}{\lambda} - 3)]} (\lambda^3 - 1)}{\lambda^3} \quad \epsilon = \lambda - 1$$

Reduced Relaxation Function (RRF)

$$G(t) = G_\infty + \sum_{i=1}^4 G_i \cdot e^{-\frac{t}{\tau_i}}$$

Constraints

$$1 = G_\infty + \sum_{i=1}^4 G_i \quad 0 \leq G_i \leq 1$$

$\tau_1 = 1\text{ms}$
 $\tau_2 = 10\text{ms}$
 $\tau_3 = 100\text{ms}$
 $\tau_4 = 1000\text{ms}$

Ramp and Hold Tests

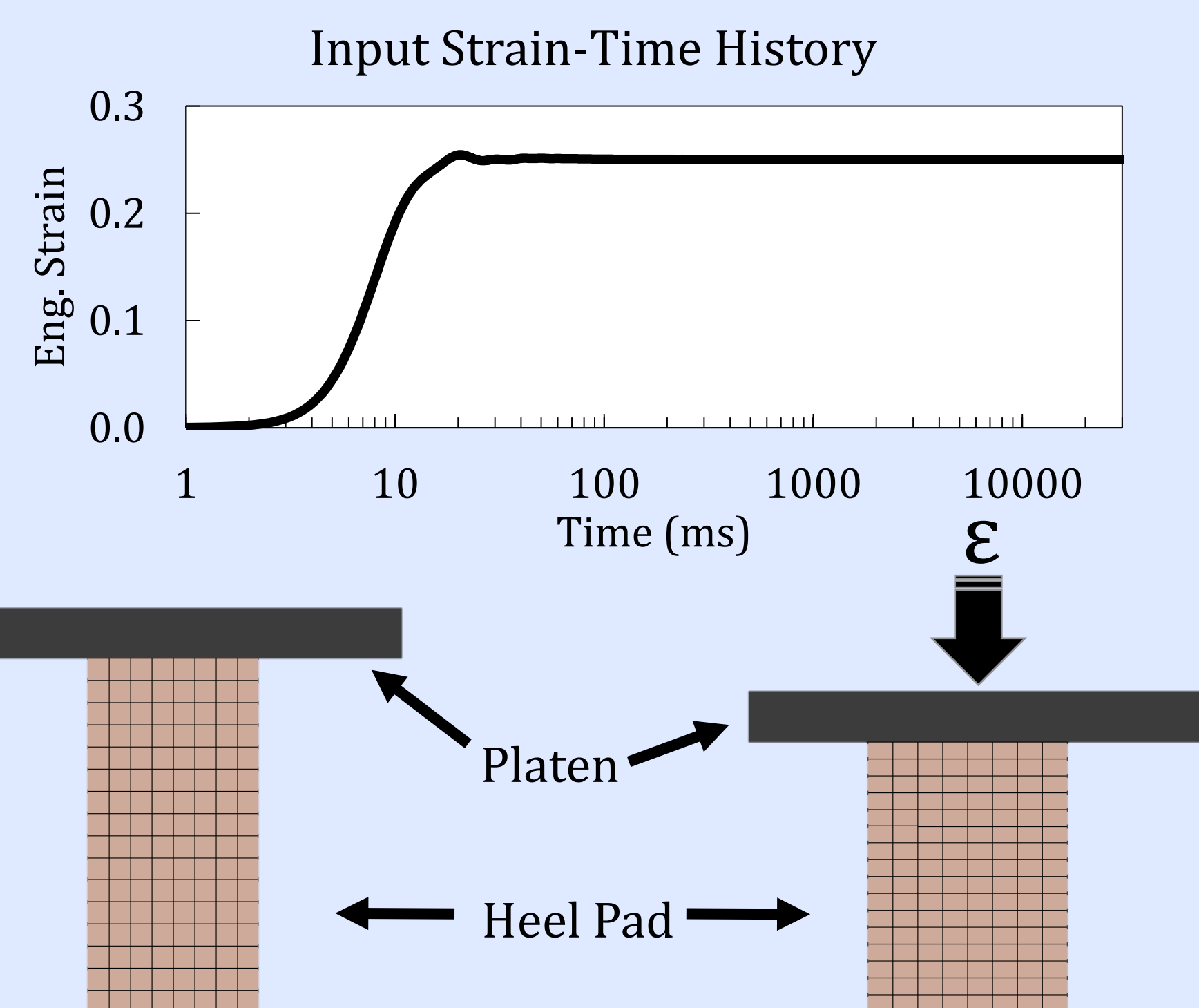
- 17 Cubic samples
- $\epsilon = (10-45)\%$ Compression
- Avg./Peak $\dot{\epsilon} = 25\text{s}^{-1}/60\text{s}^{-1}$

Uniaxial Compression

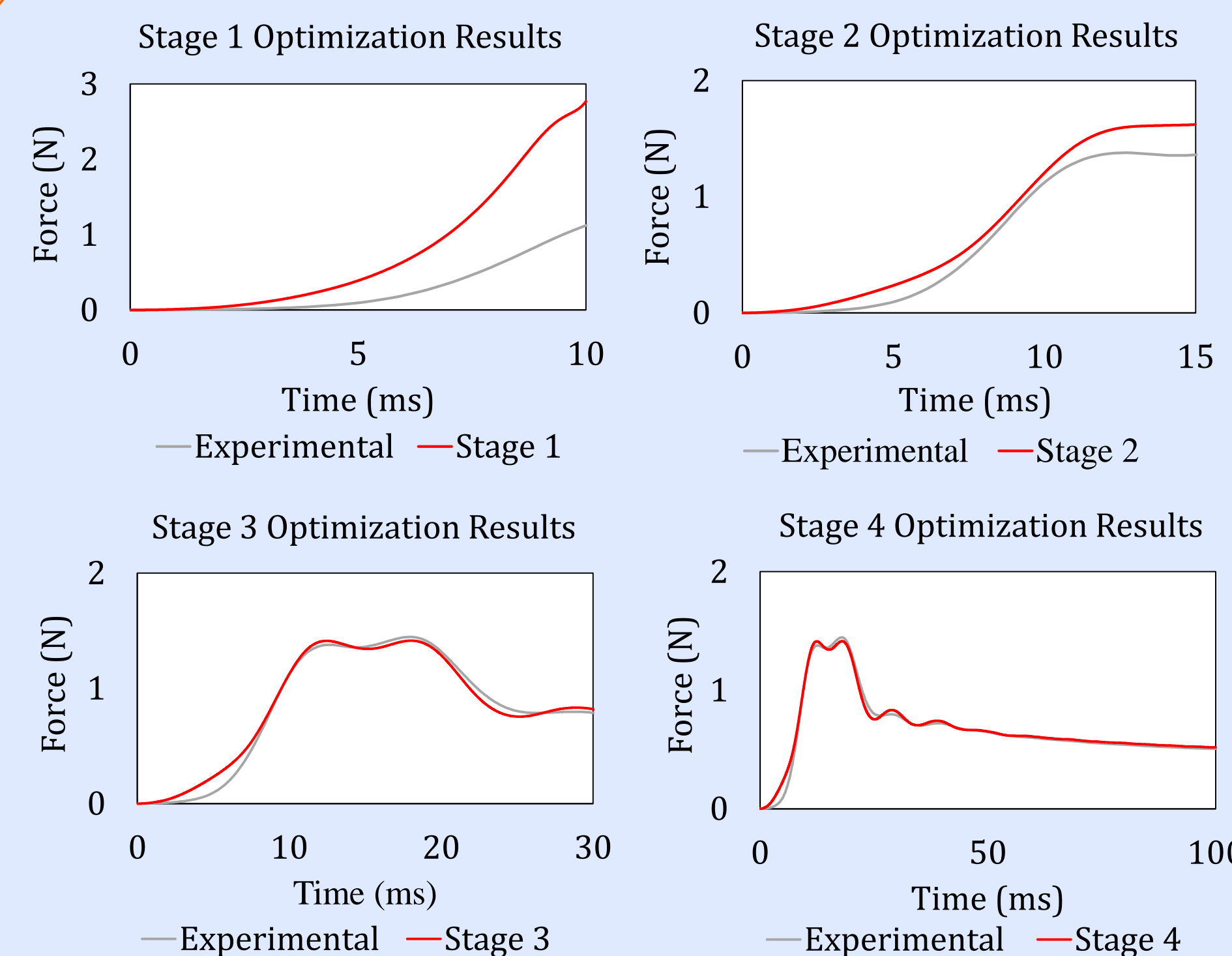
- Experimental displacement-time history prescribed to platen

FE Model

- Heel pad quarter-model
- 1024 hexahedral elements
- Incompressible material
- Platen
- Single rigid hexahedral element

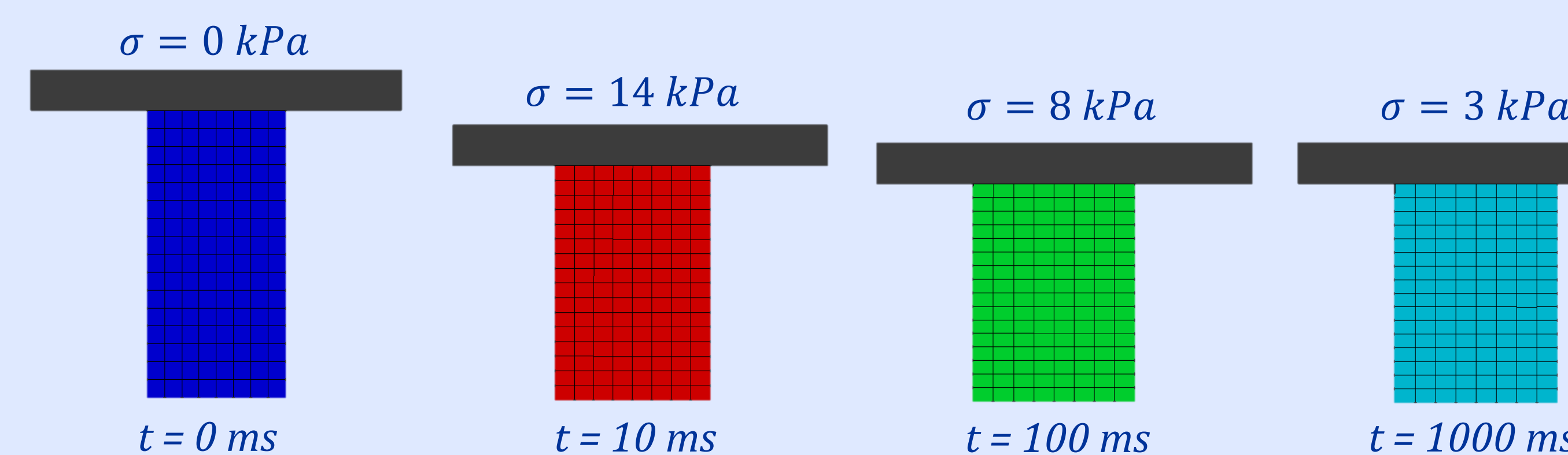


Results

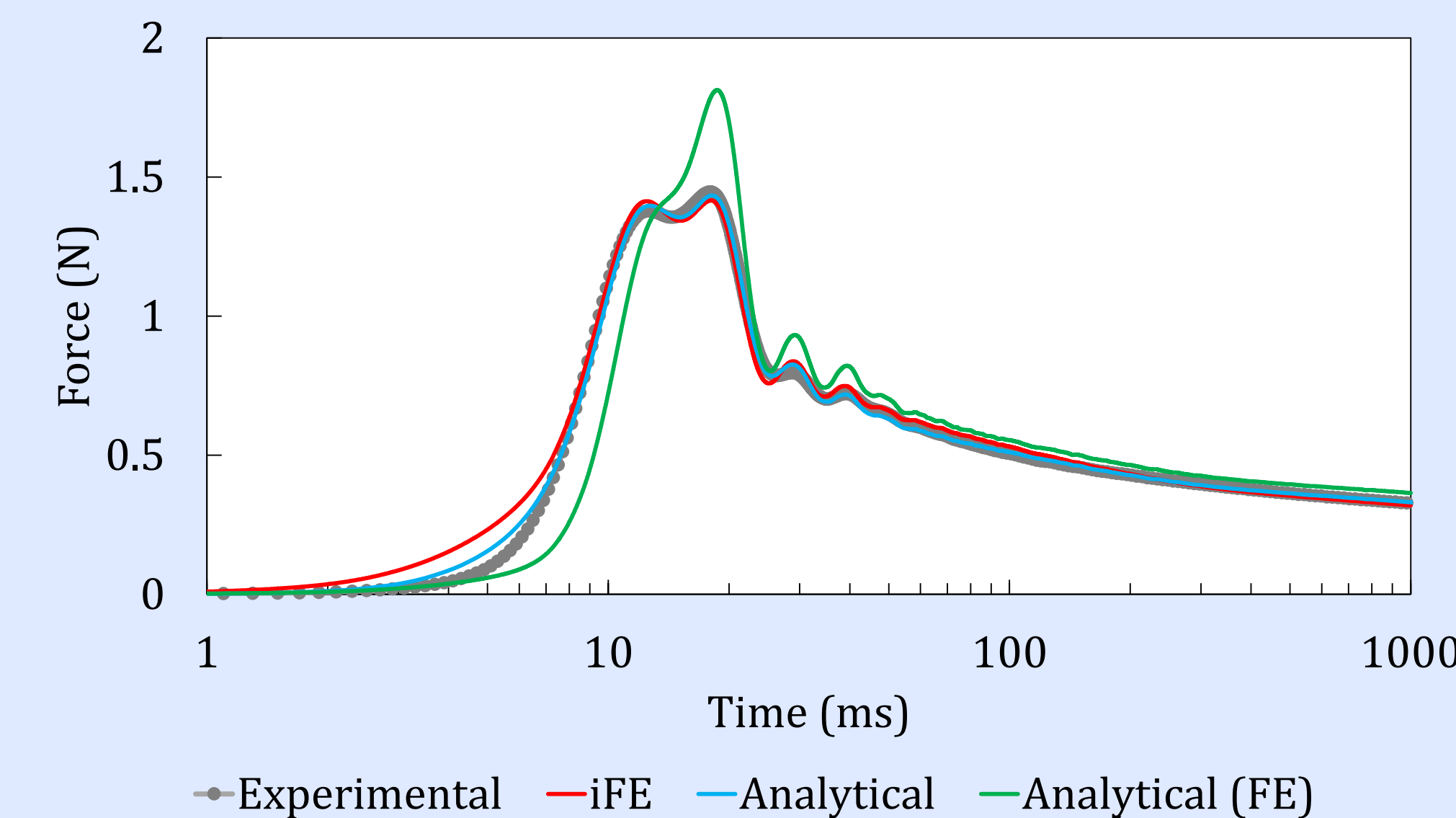


iFE Material Model Convergence

Stage	μ_0	γ	G_1	G_2	G_3	G_4	G_{inf}
1	31.7	3	0	0	0	0	1
2	34.1	4.8	0.8	0	0	0	0.2
3	34.9	5	0.8041	0.1036	0	0	0.0923
4	34.9	5	0.8041	0.0995	0.0492	0	0.0473
5	34.9	5	0.8041	0.0995	0.0437	0.018	0.0347



FE Force Response with iFE and Analytically Derived Material Models



- ❖ Model accuracy assessed using the sum of squared error (SSE).

$$SSE = \sum_{i=1}^N (y_i - f(x_i))^2$$

- ❖ The analytically determined constitutive model was applied to the FE heel pad model under identical boundary conditions

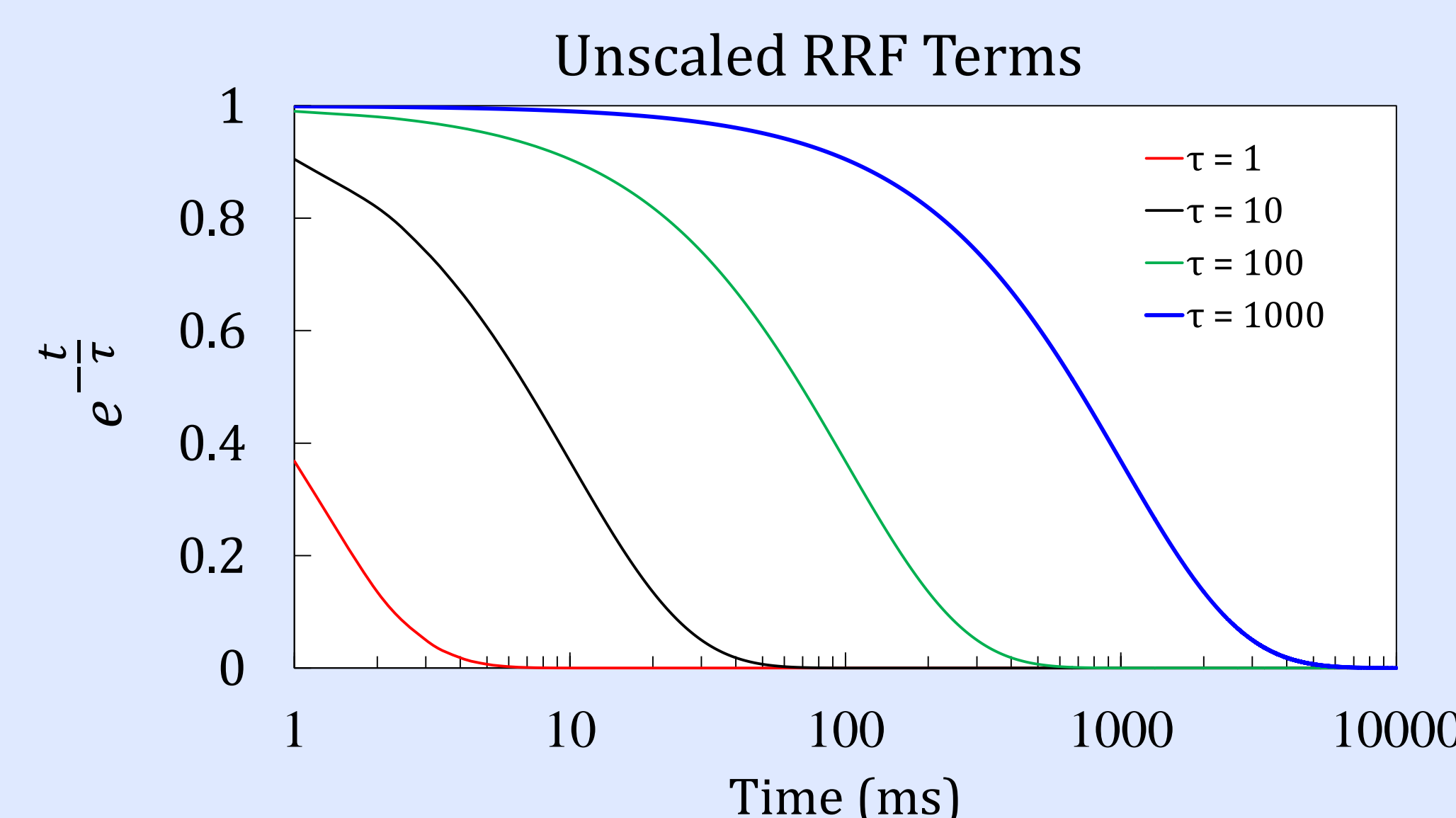
Analytical and iFE Material Models

QLV Model Coefficients	Analytical Values [1]	iFE Values
μ_0 (kPa)	31.43	35.00
γ	10.86	5.000
$G_1(\tau_1 = 1\text{ms})$	0.7246	0.8041
$G_2(\tau_2 = 10\text{ms})$	0.1633	0.0995
$G_3(\tau_3 = 100\text{ms})$	0.0454	0.0437
$G_4(\tau_4 = 1000\text{ms})$	0.0179	0.0180
$G_\infty(\tau \rightarrow \infty)$	0.0355	0.0347
SSE	0.7378	0.6428

Methods

Multi-Stage iFE

- Stage 1:** The IER parameters (μ_0, γ) are optimized over a short duration simulation. Assumed no relaxation and all $G_i = 0$. $t = 10$ ms.
- Stage 2:** The first RRF parameter (G_1) is included in the optimization and all other $G_i = 0$. $t = 15$ ms.
- Stage 3:** The second RRF parameter (G_2) is included in the optimization and all other $G_i = 0$. $t = 30$ ms.
- Stage 4:** The third RRF parameter (G_3) is included in the optimization and all other $G_i = 0$. $t = 100$ ms.
- Stage 5:** The final RRF parameter (G_4) is included in the optimization. $t = 1000$ ms.



Discussion

- ❖ The multi-stage QLV method provided a fit to the experimental data for heel pad comparable to those previously generated using an analytical method [1]
- ❖ When the analytically-derived QLV constitutive model was applied to the FE model, the force response was over-predicted.
- ❖ The multi-staged approach reduced the computational cost and the SSE by factors of 2.3 and 4.8, respectively when compared to the single-stage optimization

Fit and CPU Time for iFE Methods

iFE Method	SSE	Calculation Time (hrs)
Traditional	3.1*	17.8*
Multi-Stage	0.6	7.57

* Varied significantly with accuracy of initial guesses. Best case reported.

- ❖ The multi-stage iFE method was sensitive to initial guesses for the model parameters

Future Work

- ❖ Perform iFE analysis on remaining cases to establish a characteristic model and statistical significance.
- ❖ Implement the iFE heel pad characteristic model into the GHBM leg and assess its performance in high-rate axial compression of the tibia [1,2].
- ❖ Validate the heel pad characteristic model in other loading conditions (i.e. cyclic) to further assess the robustness of the multi-stage iFE method.

Conclusions

1. A multi-stage inverse Finite Element method was developed and applied to a heel pad FE model to determine its QLV constitutive model.
2. An analytical method provided a similar fit to the iFE method, however, performed poorly when implemented into the FE heel pad model
3. The multi-stage iFE method provided a better fit to the experimental data than the traditional iFE method, with less computational cost
4. The novel multi-stage iFE method can be applied to any viscoelastic material that exhibits a strain-dependent initial elastic response and time-dependent stress relaxation

References

- [1] Gabler, Lee F., Panzer, Matthew B., Salzar, Robert S. High-Rate Mechanical Properties of Human Heel Pad for Simulation of a Blast Loading Condition. *Proc. Int. Res. Counc. Biomech. Impact IRCOBI* 796-80 (2014).
- [2] Henderson K, Bailey A, Christopher J, Brozoski F, Salzar R. Biomechanical response of the lower leg under high rate loading. *Proceedings of IRCOBI Conference*, 2013, Gothenburg, Sweden.
- [3] Fung YC, Biomechanics: Mechanical Properties of Living Tissues, 277-280, Springer, New York, NY, 1993.