A Statistical Surrogate-Based Bayesian Approach to Calculate Brain Injury Criteria

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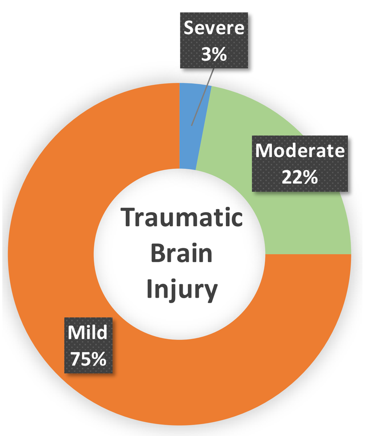
Introduction/Motivation

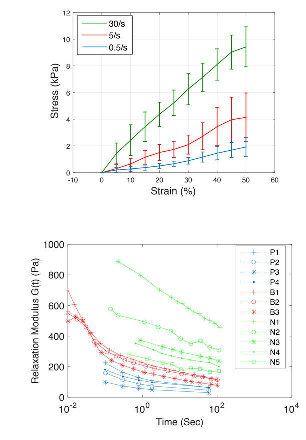
TRAUMATIC BRAIN INJURY

- Traumatic brain injury (TBI) is one of the major cause of death and disability in U.S.A.
- Approximately, 1.7 million people suffer from TBI each year with about 50000 deaths in U.S.
- Moderate and severe TBI causes focal injuries like skull fracture, hemorrage and can be detected by X-ray CT and MRI.
- Mild TBI causes diffused injuries and cannot be detected using X-ray CT and MRI.
- Mild TBI diagnosis is primarily based on neurocognitive assessments.
- Computational models of the human head are extensively used to study mild TBI.



SOFT TISSUE CONSTITUTIVE MODELS

- Large variability in the observed experimental response.
- The estimated constitutive model parameters - show a large variability.
- Constitutive models are themselves phenomenological.
- Brain tissue constitutive models are incorporated into finite element models used to study TBI.
- Calibrated using simple uniaxial/ shear experiments.



Constitutive Modeling

- Brain tissue is assumed to be an isotropic and nonlinear visco-hyperelastic.
- The long-term viscoelasticity, the short-term viscoelasticity, and hyperelasticity contributions to the mechanical response of the brain tissue are modeled separately (Pioletti et al.)

$$\sigma(t) = \sigma_e(\mathbf{C}(t)) + \sigma_v(\dot{\mathbf{C}}(t), \mathbf{C}(t)) + \mathbf{F}(t)\int_{\delta}^{\infty} \mathcal{G}(t-s, s, \mathbf{C}(s))\, ds\, \mathbf{F}(t)^T$$

$$\sigma_e(\mathbf{C}(t)) = 2\mathbf{F}(t)\frac{\partial \Psi_e}{\partial \mathbf{C}}\mathbf{F}(t)^T \qquad \sigma_v(\dot{\mathbf{C}}(t), \mathbf{C}(t)) = 2\mathbf{F}(t)\frac{\partial \Psi_v}{\partial \dot{\mathbf{C}}}\mathbf{F}(t)^T \qquad \sigma_v^l(\mathbf{C}(t), t-s) = \int_{\delta}^{\infty} \dot{\mathcal{G}}(t-s)\, \sigma_e(\mathbf{C}(t))$$

$$\Psi_e = B_1(\exp(B_2(I_1-3)) - 1) \qquad \Psi_v = \eta J_2(I_1 - 3) \qquad \mathcal{G}(s) = \left(G_\infty + \sum_{i=1}^{3} G_i e^{(-s/\tau_i)}\right)$$

Bayesian Framework for Calibration

$$f(\boldsymbol{\theta}|\boldsymbol{m}) = \boldsymbol{d} + \boldsymbol{\eta}$$

- $f(\boldsymbol{\theta}|\boldsymbol{m})$ is the model form.
- $\boldsymbol{d} \in D$ is the hypothetical true data.
- $\boldsymbol{\eta}$ is the **noise**.

Using Bayes Theorem:

$$\underbrace{P(\boldsymbol{\theta}|\mathbf{d}, m, I)}_{\text{Posterior}} = \frac{\overbrace{P(\mathbf{d}|\boldsymbol{\theta}, m, I)}^{\text{Likelihood}}\overbrace{P(\boldsymbol{\theta}|m, I)}^{\text{Prior}}}{\underbrace{P(\mathbf{d}|m, I)}_{\text{Evidence}}}$$

PRIOR PDF OF THE PARAMETERS

The interval $[a, b]$ is chosen based on prior knowledge and a simulation study.

NOISE: ADDITIVE GAUSSIAN

Likelihood : Gaussian $(0, \sigma)$

$$P(\mathbf{d}|\boldsymbol{\theta}, m, I) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i{}^2}} \exp\left[-\frac{(f(\boldsymbol{\theta}|\boldsymbol{m}) - d_i)^2}{2\sigma_i{}^2}\right]$$

SAMPLING ALGORITHM

Used to obtain the posterior and evidence.

NESTED MONTE CARLO SAMPLING based algorithm MULTINEST.

NESTED MONTE CARLO SAMPLING

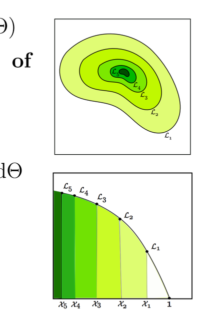
Goal: Calculate $P(\mathbf{d}|m, I) = \int_{\boldsymbol{\theta}} P(\mathbf{d}|\boldsymbol{\theta}, m, I)P(\boldsymbol{\theta}|m, I)\, d\boldsymbol{\theta}$

Prior distribution: $\pi(\Theta)$

Differential element of prior "mass":
$\rightarrow dX = \pi(\Theta)d\Theta$

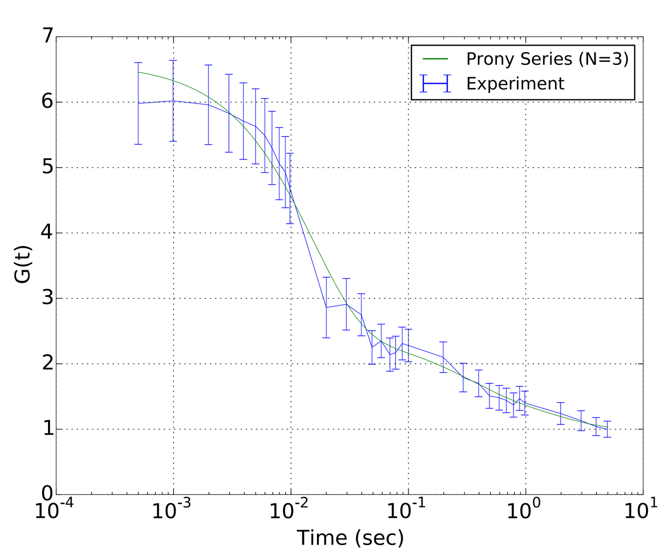
Prior volume :
$\rightarrow X(\lambda) = \int_{\{\theta:\mathcal{L}(\Theta)>\lambda\}} \pi(\Theta)d\Theta$

Evidence :
$\rightarrow \mathcal{Z} = \int_0^\infty X(\lambda)d\lambda$
$\rightarrow \mathcal{L}(X(\lambda)) = \lambda$
$\rightarrow \mathcal{Z} = \int_0^1 \mathcal{L}(X)dX$



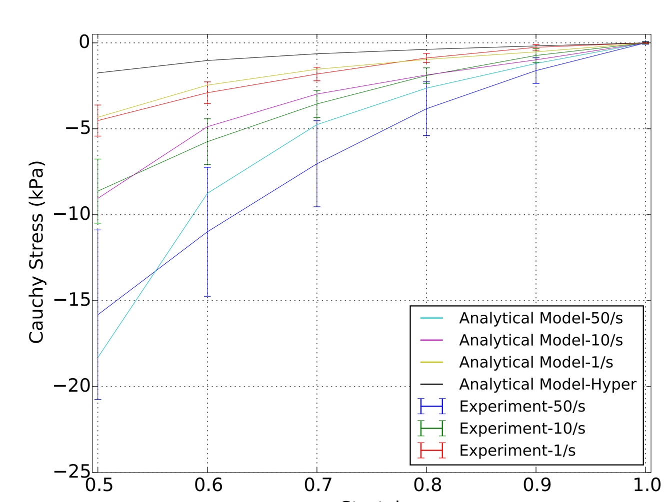
Calibration process - Stochastic Nonlinear Visco-Hyperelastic Model

- Cylindrical specimens d=22mm, l=14mm
- Relaxation test in compression: Strain rate of 50 s^{-1}
- Max nominal strain of 20,30,40,50,60,70 %



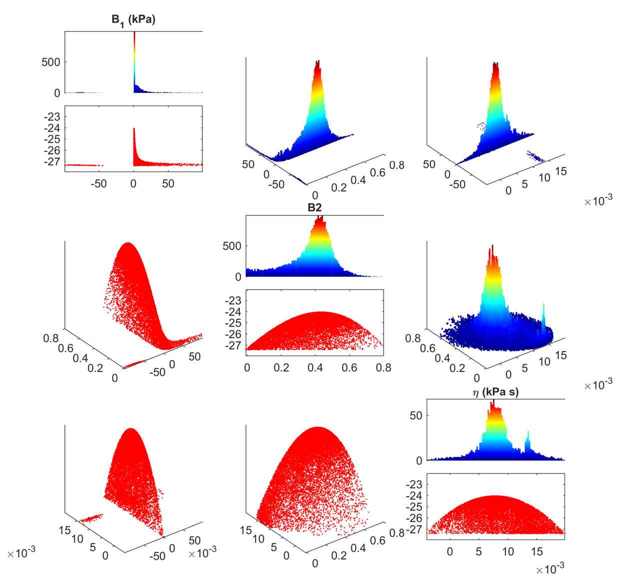
G_1	$\tau_1(s)$	G_2	$\tau_2(s)$	G_3	$\tau_3(s)$
4.168	1.512 E-02	0.828	2.833 E-01	0.606	1.732

- Uniaxial unconfined compression test
- Strain rate of 1 s^{-1}, 10 s^{-1},50 s^{-1}
- Compare the response constitutive model with MLE of the parameters to the experimental data



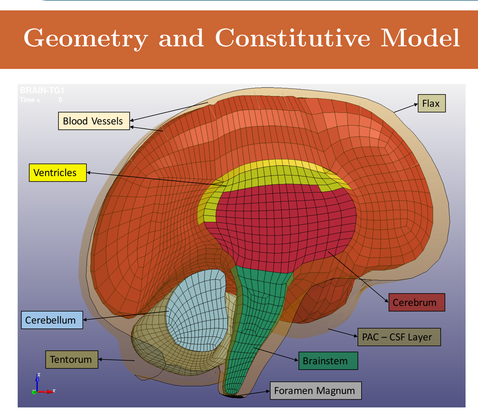
$B_1(kPa)$	B_2	$\eta(kPa\,s)$
0.675	0.431	0.793 E-02

- Distribution of the parameters $B_1(kPa)$, B_2 , $\eta(kPa\,s)$
- Obtained using Bayesian calibration



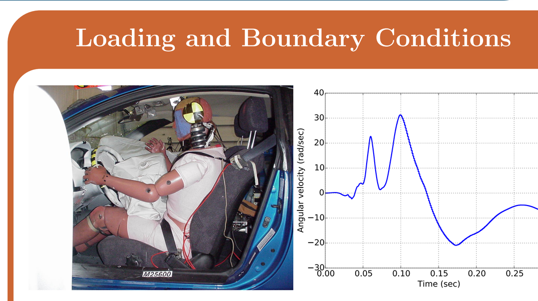
Deterministic Computational Model

Geometry and Constitutive Model



- SIMon FE head model
- 45875 elements
- Cerebrum, cerebellum, and brain stem - nonlinear visco-hyperelastic
- UMAT in LS-Dyna
- Used MLE values of parameters

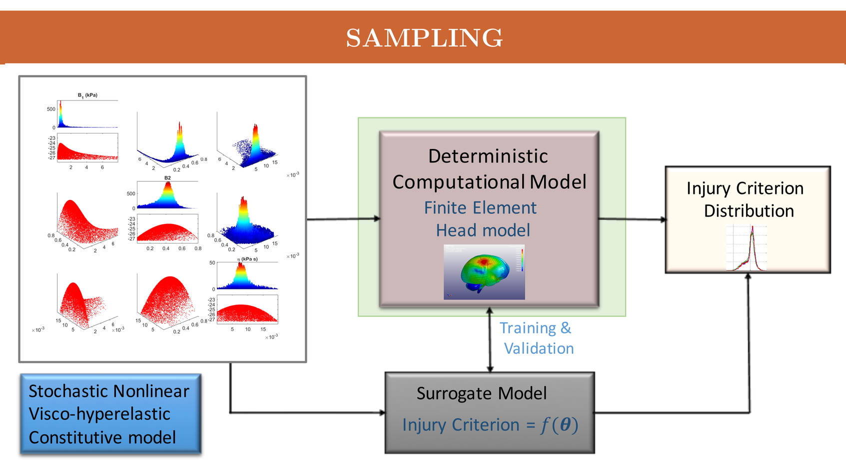
Loading and Boundary Conditions



- A free boundary condition - simulate impact
- Time duration of the impact - too short for the neck to influence kinematics of the head response
- Choose only angular velocity in sagittal plane
- Impact duration of 0.115 sec
- Each simulation takes 60 minutes on 60 processors

Statistical Surrogate Model for the Computational Model

SAMPLING



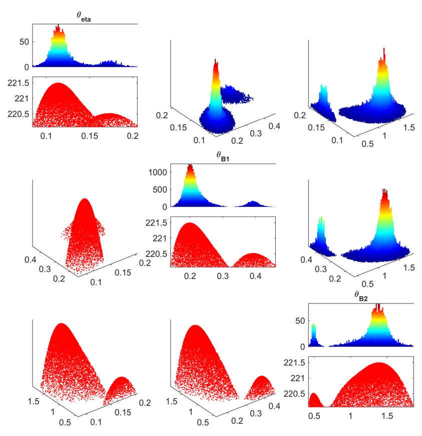
- A finite element simulation for every sample (22000) - not feasible computationally
- 104 samples - parameter space - Latin hypercube sampling - LS-OPT
- 90 - training , 14 - validating the surrogate model
- A finite element simulation run for each of the 104 samples - Injury criterion calculated

GAUSSIAN PROCESS MODEL

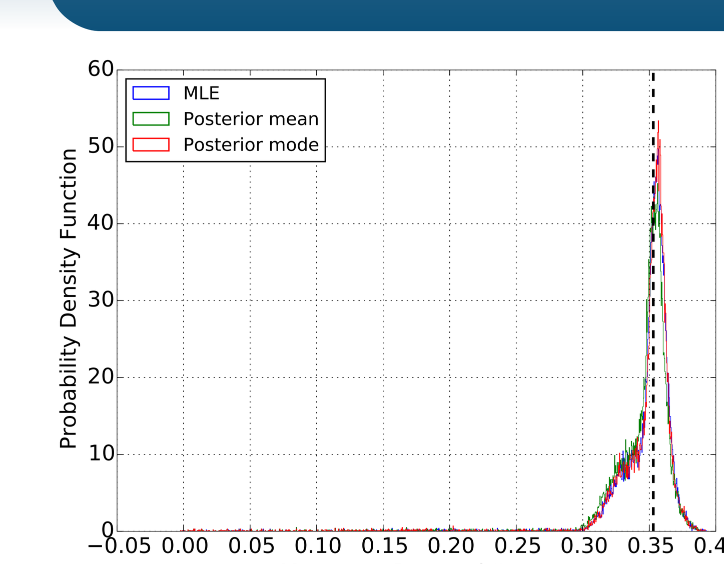
- Stationary spatial process is represented by

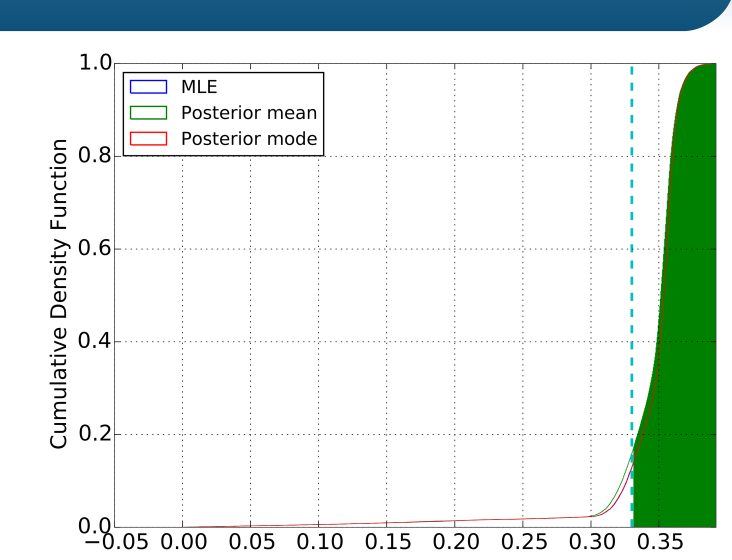
$$\mathbf{Y}(\boldsymbol{\theta}) = \boldsymbol{\mu}(\boldsymbol{\theta}) + \mathbf{w}(\boldsymbol{\theta}) + \boldsymbol{\epsilon}(\boldsymbol{\theta})$$

- $\boldsymbol{\theta}$: parameter space, $\mathbf{Y}(\boldsymbol{\theta})$: injury criterion
- $\boldsymbol{\mu}(\boldsymbol{\theta}) = \mathbf{X}^T(\boldsymbol{\theta})\boldsymbol{\beta}$, $\mathbf{w}(\boldsymbol{\theta}) = \mathcal{N}\left(\mathbf{0}, \mathcal{C}(\boldsymbol{\phi}, \sigma^2)\right)$
- $\boldsymbol{\epsilon}(\boldsymbol{\theta}) = \mathcal{N}\left(\mathbf{0}, \tau^2 I\right)$
- Obtain the posterior probability $P(\{\boldsymbol{\beta}, \boldsymbol{\phi}, \sigma^2\}|\mathbf{Y})$



Brain Injury Criteria Distribution





- Maximum principal strain is selected as the injury criterion.
- Uncertainty in the material parameters is propagated to the injury criterion.
- Considering the injury threshold to be 0.33 - The probability that the mild TBI injury occurs for this loading is 85%

References

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3. S. Madireddy, B. Sista, K. Vemaganti, "Bayesian calibration of hyperelastic constitutive models of soft tissue", Journal of Mechanical Behavior of Biomedical Materials., 59 (2016) 108-127.
4. DP. Pioletti and LR. Rakotomanana, "Non-linear viscoelastic laws for soft biological tissues", European Journal of Mechanics A-Solids, 19(2000), 749?759.

Summary & Conclusions

- Bayesian framework for calibration takes the experimental uncertainty into consideration to obtain a distribution of parameters.
- Experimental data used covers the typical strain rates experienced during impact loads.
- Developed a stochastic nonlinear visco-hyperelastic model that describes the experimental data and its uncertainty.
- A finite element computational model (SIMon) is used to simulate the injury load in vehicle crash test.
- The surrogate model based approach enables us to calculate the probability that an injury tolerance is reached for a given impact loading.
- This probabilistic method can be used to further simulate injuries and calculate various injury criteria with higher fidelity.

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