Modelling Collagen Fibre Failure as a Function of Strain Rate

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Introduction:

Soft tissue neck injury—also known as whiplash—is the most common motor vehicle injury treated in U.S. hospital emergency departments [1]. Mathematical models are critical tools for investigating soft tissue injury in simulations above human injury thresholds. These models depend on the accurate characterization of the mechanical response of collagenous-tissue [2]. However, to date, little attention has been given to modelling these tissues as they are failing. Thus, the purpose of this investigation was to use a collagen distribution model to characterize the mechanical properties of isolated rat tail tendons, as an *in-vitro* collagenous tissue model, while strained until failure and assess the influence of strain rate on such mechanical properties. The time-evolution of a population of collagen fibres can be modelled using the differential equation [3]:

$$\frac{\partial \rho}{\partial t} - v\frac{\partial \rho}{\partial \varepsilon} = -B(\varepsilon, t, \rho) \tag{1}$$

Where $\rho(\varepsilon,\ t)$ is the distribution of strain among the collagen fibres such that $\rho(\varepsilon,\ t)\Delta\varepsilon$ represents the proportion of collagen fibres strained by ε at time t; v is the strain rate; and $B(\varepsilon,\ t,\ \rho)$ is a function describing the rate of breaking. We tested a breaking function of the form:

$$B(\varepsilon, t, \rho) = (b_0 + b_1(\varepsilon - \varepsilon_0))\Theta(\varepsilon - \varepsilon_0)\rho(\varepsilon, t) \tag{2}$$

Where $\Theta(\cdot)$ is the Heaviside step function. This supposes that the rate at which collagen fibres break is linear in the magnitude they have been strained beyond some threshold, ε_0. This investigation determined the parameters of the initial distribution $\rho(\varepsilon, 0)$, and how the three breaking function parameters (ε_0, b_0 and b_1) depend on the strain rate. In the above model, stress is calculated by integrating over ε:

$$\sigma(t) = \int_0^\infty E\varepsilon\, \rho(\varepsilon, t) d\varepsilon \tag{3}$$

E is the tangent modulus in the linear region of the stress strain curve.

Methods

Fifty rat tail tendon specimens were excised from a single source for initial verification of the model. Each specimen was mounted uniaxially in the longitudinal direction, in a displacement controlled biaxial biological tissue testing system (Biotester, Cellscale, Waterloo Ontario). Specimens underwent a cyclic preload, 10% strain at 1 Hz for 20 cycles [4] before being strained to failure at one of five strain rates: 0.01, 0.05, 0.10, 0.15, and 0.20 s^{-1}. Force and displacement—renormalized to engineering stress and strain—were sampled at 100 Hz.

The tangent modulus in the linear portion of the stress strain curve, as well as two parameters representing the average slack strain (μ) and standard deviation of slack strain (σ) were obtained [5]. The remaining parameters—b_0, b_1 and ε_0—were obtained from least-squares. These six parameters were linearly regressed against strain rate to assess the influence of strain rate on such parameters.

Results

Of the six parameters, μ, σ and b_1 were found to vary significantly with strain-rate. In most cases (34 of 50), b_0 was found to be identically zero. The average root-mean-square error was 0.95 MPa (Range: 0.17 – 3.97 MPa), despite a large variability in parameter estimates (Fig 1).

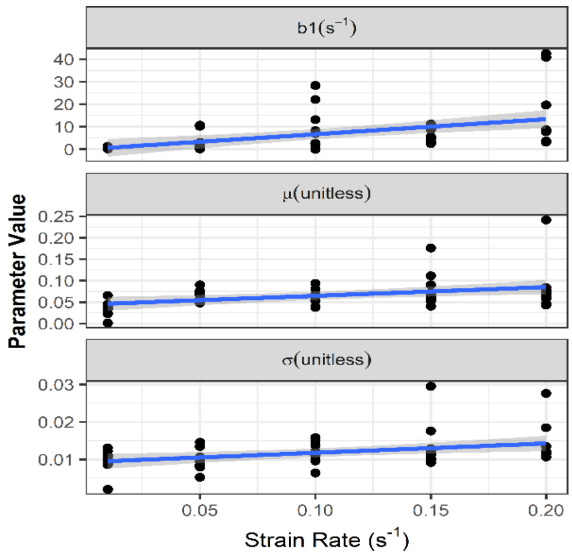

Figure 1: Parameters b_1, μ and σ as a function of strain rate.

Conclusions

Results from this investigation suggest that for positive strain rates up to 0.20 s^{-1}, there is evidence to support using a breaking function of the form (Eq. 4):

$$B(x, t, \rho) = b_1 v(t)(x - x_0)\Theta(x - x_0)\rho(x, t) \quad (4)$$

References:

[1] Quinlan et al. (2004) *Accid Anal Prev*. **36**, 21–27.
[2] Lucas et al. (2008). *Acta Biomater*. **4**(1), 117-125.
[3] Barrett & Callaghan, (2017). *J. Biomech. **61**, 11-17.*
[4] Bass et al. (2007). *Spine.* **32**(1), E7-E13.
[5] Barrett & Callaghan, (2018). *J. Biomech*. **66**, 175-179.